DELHI PUBLIC SCHOOL, BHILAI

MODEL PAPER-2024-'25 — **Time : 3 Hrs.**

Class : XII — **SUBJECT : MATHEMATICS (041)** — **Max. Marks : 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory
2. Section – A carries 20 marks weightage, section – B carries 10 marks weightage, section – C carries 18 marks weightage, Section – D carries 20 marks weightage and section – E carries 3 case-based with total weightage of 12 marks.
3. Section – A comprises 20 MCQs of 1 mark each.
4. Section – B comprises 5 VSA type questions of 2 marks each.
5. Section – C comprises 6 SA type of questions of 3 marks each
6. Section – D comprises 4 LA type of questions of 5 marks each.
7. SECTION – E It has 3 case studies. Each case study comprises 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in two case-study.
8. Internal choice is provided in 2 questions in section – B, 2 questions in section – C, 2 questions in section – D. You have to attempt only one of the alternatives in all such questions.

SECTION A

1. The derivative of the function $f(x) = x|x|\ at\ x = 0$, is
(a) 1 (b) 2 (c) -2 (d) 0

2. For matrix $A = \begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, $(adjA)'$ is equal to
(a) $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$ (b) $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} 7 & -5 \\ 11 & 2 \end{bmatrix}$.

3. If for the matrix $A = \begin{bmatrix} \alpha & -3 \\ -3 & \alpha \end{bmatrix}$, $|A|^3 = 343$, then the value of α is
(a) ± 3 (b) -3 (c) ± 4 (d) 1

4. Given that A is a square matrix of order 3 and $|A| = 8$, then $|adjA|$ is equal to
(a) 1 (b) 64 (c) 2 (d) 8

5. Let A be non-singular matrix of order n, then $|adj(adjA)|$ is equal to
(a) $|A|^{n-1}$ (b) $|A|^{n-2}$ (c) $|A|^{(n-1)^2}$ (d) $|A|^{n^2}$

6. Let A and B be 3×3 matrices such that $A' = -A$ and $B' = B$then matrix $\lambda AB + 3BA$ is a skew symmetric matrix for. (a) 1 (b) 64 (c) 2 (d) 8

7. The function $f(x) = \frac{4-x^2}{4x-x^3}$, is
(a) discontinuous at only one point (b) discontinuous exactly at two points (c) discontinuous exactly at three point (d) none of these

8. The function $f(x) = 2x^3 - 3x^2 - 12x + 4$, has
(a) Two points of local maximum(b) two points of local minimum (c) one maximum and one minimum (d) no maximum and no minimum

9. $\int \frac{1}{cos^2x.sin^2x} dx$ equals to
(a) $tan\,x + \cot x + c$ (b) $tan\,x - \cot x + c$ (c) $tan\,x \cot x + c$ (d) $tan\,x - \cot 2x + c$.

10. If $\int_{-1}^{1} \frac{|x-2|}{x-2} dx, x \neq 2$, is equal to (a) 1 (b) -1 (c)) 2 (d) -2

11. The area bounded by the curve $y = x^3$, the x-axis and the ordinates $x = -2\ and\ x - 1$, is
(a) $\frac{15}{4}$ (b) -9 (c) $-\frac{15}{4}$ (d) $\frac{17}{4}$

12. Integrating factor of $x\frac{dy}{dx} - y = x^4 - 3x$, is
(a) x (b) $logx$ (c) $\frac{1}{x}$ (d) $-x$

13. The solution differential equation $\frac{dx}{x} + \frac{dy}{y} = 0$ $f'\left(\frac{\pi}{4}\right)$ is equal to
(a) $\frac{1}{x} + \frac{1}{y} = C$ (b) $logx - logy = C$ (c) $xy = C$ (d) $x + y = C$

14. Let The scalar product of $\hat{\imath} + \hat{\jmath} + \hat{k}$ with a unit vector along the sum of the vectors $\widehat{2\imath} + \widehat{4\jmath} - 5\hat{k}$ and $\widehat{\lambda\imath} + 2\hat{\jmath} + \widehat{3k}$ then the value of λ is
(a) 1 (b) 3 (c) 4 (d) 2

15. If $\vec{a}$ is non zero vector of modulus a and λ is anon zero scalar and $\lambda\vec{a}$ is a unit vector then
(a) $\lambda = \pm 1$ (b) $a = |\lambda|$ (c) $a = \frac{1}{|\lambda|}$ (d).None of the above

16. The objective function $Z = ax + by$ of an LPP has the maximum value 42 at (4,6) and the minimum value 19 at (3,2), which of the following is true? (a) $a = 9, b = 1$ (b) $a = 5, b = 2$ (c) $a = 3, b = 5$ (d) $a = 5, b = 3$

17. The number of feasible solutions of the linear programming problem given as, Maximize $Z = 15x + 30y$, subject to : $3x + y \leq 12, x + 2y \leq 10, x, y \geq 0$, is

(a) 1 (b) 2 (c) 3 (d) infinite

18. If A and B are two independent events with $P(A) = \frac{3}{5}$ and $P(B) = \frac{4}{9}$, then $P(A' \cap B')$, equals

(a) $\frac{4}{15}$ (b) $\frac{8}{45}$ (c) $\frac{1}{3}$ (d) $\frac{2}{9}$

For questions 19 and 20, two statements are given – one labeled Assertion (A) and the other labeled Reason (R), select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below:

(a) Both A and R are true and R is the correct explanation of the assertion
(b) Both A and R are true and R is not the correct explanation of the assertion
(c) A is true, but R is false
(d) A is false, but R is true

19. Assertion (A) : $sin^{-1}(-x) = -sin^{-1}x, \forall x \in R$
Reason (R) : $sec^{-1}(-x) = -sec^{-1}x, x \in R - (-1,1)$

20. Assertion (A) : Let A and B are two finite sets having m and n elements respectively then the number of functions from A to B is n^m
Reason (R) :Every function from A to B is a subset of $A \times B$

SECTION B

21. If $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

OR

Find the value of λ, so that the following lines are perpendicular to each other: $\frac{x-5}{5\lambda+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $\frac{x}{1} = \frac{2y+1}{4\lambda} = \frac{1-z}{-3}$

22. Evaluate:- $tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) + cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + tan^{-1}\left(\sin\left(\frac{-\pi}{2}\right)\right)$

23. If $f(x) = |\tan 2x|$, then find the value of $f'\left(\frac{\pi}{4}\right)$

OR

Check the continuity of the function $f(x) = \begin{cases} \frac{1-\cos x}{x^2}, \text{if } x \neq 0 \\ \frac{1}{2}, \text{if } x = 0 \end{cases}$

24. Find the point on the curve $y^2 = 8x$ for which the abscissa and the ordinate change at the same rate?

25. Using vector method, find the area of the triangle formed by the points A(1,2,3), B(2, −1,4) and C(4,5, −1).

SECTION C

26. Find the point on the curve $4y = x^2$ which is nearest to the point $(-1,2)$.

27. An inverted cone has a depth of 10 cm and a base of radius 5 cm. Water is poured into it at the rate of 1.5 c.c.per minute. Find the rate at which the level of water is rising when the depth is 4 cm

28. Evaluate $\int e^x \frac{(x^2+1)}{(x+1)^2}\, dx$

29. Find the shortest distance between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x+2}{1} = \frac{y-3}{2} = \frac{z+1}{4}$,

OR

Let $\vec{a}, \vec{b}$ and $\vec{c}$ are three vector with magnitudes 3,4 and 5 respectively such that one vector is perpendicular to the sum of other two, then find the value of $|\vec{a} + \vec{b} + \vec{c}|$

30. Maximize: - $Z = 5x + 2y$, subject to :- $-2x - 3y \leq -6$, :- $x - 2y \leq 2$, $3x + 2y \leq 12$, $-3x + 2y \leq 3$, $x, y \geq 0$

31. A man is known to speak truth 3 out of 5 times. He throws a die and reports that it is 4. Find the probability that it is actually a 4.

OR

An urn contains 3 red and 5 black balls. A ball is drawn at random; its colour is noted and returned to the urn. Moreover, 2 additional balls of the colour noted down, are put in the urn and then two balls are drawn at random (without replacement) from the urn. Find the probability that both the balls drawn are of red colour.

SECTION D

32. Use the product $\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ to solve the system of equation:- $x - y = 3, 2x + 3y + 4z = 17,\ y + 2z = 7$

33. If $y = \{log(x + \sqrt{x^2 + 1})\}^2$ then show that $(1 + x^2)y_2 + xy_1 = 2$

OR

If $x = a(\theta - \sin\theta)$ and $y = a(1 + \cos\theta)$ then find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{2}$

34. Using integration, find the area of the region bounded by the curve $y = 2x - x^2$ and the straight line$y = -x$.

OR

Find the area of the region included between the curves $y = |x - 1|\ and\ y = -|x - 1| + 1$

35. Find the vector and Cartesian equation of the line which passes through the point (1,2,-4) and is perpendicular to two lines $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

SECTION -E

36. Case Study – 1 An organization conducted bike race under 2 different categories-boys and girls. In all there were 250 participants. Among all of them finally three from category-1 and two from category=2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$ and $G = \{g_1, g_2\}$, B represents the set of boys and G represents the set of girls who were selected for the final race.

Ravi decides to explore these sets for various types of relations and functions; based on the above, answer the following questions, show steps to support your answer

(i) Ravi wishes to form all the relations possible from B to G. how many such relations are possible?

(ii) Write the smallest equivalence relation on G.

(iii) Ravi defines a relation from B to B as $R_1 = \{(b_1, b_2,)(b_2, b_1,)\}$. Write the minimum ordered pairs to be added in R_1 so that it becomes reflexive but not symmetric

OR,

Ravi defines a relation from B to B as $R_1 = \{(b_1, b_2,)(b_2, b_1,)\}$. Write the minimum orderd pairs to be added in R_1 so that it becomes reflexive and symmetric but not transitive

37. Case Study – 2 Swati is to prepare a handmade gift box for her friend's marriage. For making lower part of the box, she takes a square piece of cardboard of side 20 cm.

Based on the above, answer the following questions, show steps to support your answer.

(i) If x be the length of each side of the square to be cut off from corners of the square piece of side 20 cm, find the volume of the open box formed (in terms of x) by folding up the cutting corners.

(ii) Find the value of x for which $\frac{dV}{dx} = 0$.

(iii) Find the maximum volume of the box.

OR,

Find the surface area of the box

38. Case Study-3, A shopkeeper sells three types of flower seeds, A, B and C. They are sold in the form of a mixture, where the proportion of the seeds is 4:4:2 respectively. The germination rates of the three types of seeds are 45%, 60% and 35% respectively. Based on the above, answer the following questions, show steps to support your answer

Calculate the probability that (i) A randomly chosen seed will germinate

(ii) the seed is of the type B, given that a randomly chosen seed germinates

-S.Predeep

DELHI PUBLIC SCHOOL, BHILAI

DATE :
CLASS : XII
Name : ________________

SAMPLE QUESTION PAPER 2024-'25
SUBJECT – MATHEMATICS

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Max. Marks : 80
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Section – A

01. The area of the region bounded between the line $x = 2$ and the parabola $y^2 = 8x$ is

(A) $\frac{34}{3}$ sq. units (B) $\frac{32}{3}$ sq. units (C) $\frac{16}{3}$ sq. units (D) $\frac{8}{3}$ sq. units

02. If the matrix $A = \begin{pmatrix} 0 & a & 3 \\ 2 & b & -1 \\ c & 1 & 0 \end{pmatrix}$ is skew symmetric, then the value of a, b and c are respectively:

(A) $-2, 0, 1$ (B) $2, 1, -1$ (C) $2, 0, -1$ (D) $2, 0, 1$

03. The interval in which $f(x) = x^x, x > 0$ is strictly increasing is

(A) (e, ∞) (B) $\left(\frac{1}{e}, \infty\right)$ (C) $\left(0, \frac{1}{e}\right)$ (D) $(0, \infty)$

04. If A is an invertible matrix of order 3 and $|A| = 4$, then $|adj\ (adj\ A)|$ is

(A) 64 (B) 16 (C) 256 (D) none of these

05. The degree of differential equation $\sqrt{1 + \frac{d^2y}{dx^2}} = \frac{dy}{dx} + x$ is

(A) 1 (B) 2 (C) 3 (D) not defined

06. The angle between $\vec{a}$ and $\vec{b}$ with magnitude $\sqrt{3}$ and 2 respectively and $\vec{a}.\vec{b} = \sqrt{6}$ is

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{3}$ (C) $\frac{\pi}{2}$ (D) none of these

07. If A and B are two matrices oforders ax3 and 3xb respectively such that AB exists and is of order 2×4 then (a,b) is equal to

(A) $(4, 2)$ (B) $(2, 4)$ (C) $(3, 4)$ (D) $(2, 3)$

08. Given that A and B are two events such that $P(A) = 0.6$, $P(B = 0.3$ and $P(A \cap B) = 0.2$, $P(B/A)$ is

(A) $\frac{2}{3}$ (B) $\frac{1}{5}$ (C) $\frac{1}{3}$ (D) none of these

09. If the lines $\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$ and $\frac{x-1}{3k} = \frac{5-y}{-1} = \frac{6-z}{5}$ are at right angle, then the value of k is

(A) $\frac{12}{7}$ (B) $\frac{11}{7}$ (C) $-\frac{10}{7}$ (D) $\frac{-15}{7}$

10. The unit vector perpendicular to both vectors $\hat{\imath} - 2\hat{\jmath} + 3\hat{k}$ and $\hat{\imath} + 2\hat{\jmath} - \hat{k}$ is

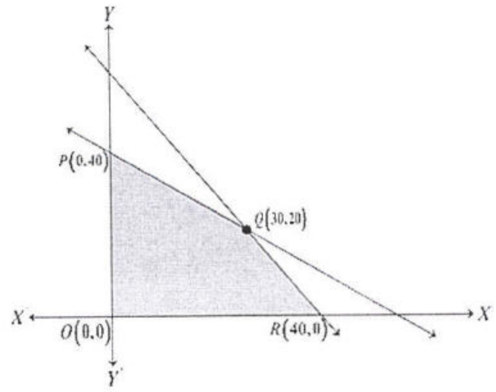
(A) $4\hat{\imath} - 4\hat{\jmath} + 4\hat{k}$ (B) $-4\hat{\imath} - 4\hat{\jmath} + 4\ k$ (C) $\frac{1}{\sqrt{3}}(-\hat{\imath} + \hat{\jmath} + \hat{k})$ (D) $\frac{1}{\sqrt{3}}(\hat{\imath} - \hat{\jmath} + \hat{k})$

11. The value of $\int_0^{\frac{\pi}{2}} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}}\ dx$

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) π (D) 0

Contd…2

12. For the linear programming problem (LPP), the objective function $z = 4x + 3y$ and the feasible region determined by a set of constraints is shown in the graph



Which of the following statements is true?
(A) Maximum value of z is at R(40, 0)
(B) (B) Maximum value of z is at Q(30, 20)
(C) Value of z at R(40, 0) is less than the value at P(0, 40)
(D) The value of z at Q(30, 20) is less than the value at R(40, 0)

13. The value of $\int \frac{xe^x}{(1+x)^2}\,dx$ is
(A) $\frac{1}{(x+1)^2}e^x + c$ (B) $-\frac{1}{(x+1)^2}e^x + c$ (C) $\frac{1}{1+x}e^x + c$ (D) $\frac{-e^x}{(1+x)}e^x + c$

14. The solution of $\frac{dy}{dx} = \frac{x}{x^2+1}$ is
(A) $y = log|x^2 + 1| + c$ (B) $2y = log|x^2 + 1| + c$
(C) $y = 2log|x^2 + 1| + c$ (D) $y^2 = log|x^2 + 1| + c$

15. The value of $tan\left\{2tan^{-1}\frac{1}{5}\right\}$ is
(A) $\pi/2$ (B) π (C) 2π (D) $\pi/4$

16. The graph of inequality $2x + 3y > 6$ is
(A) half plane that contains the origin.
(B) half plane that neither contains the origin nor the points of the line $2x + 3y = 6$.
(C) Whole XOY plane excluding the points on the line $2x + 3y = 6$.
(D) Entire XOY plane.

17. If $f(x) = \begin{Bmatrix} x^2 + 3x + a & , & x \le 1 \\ bx + 2 & , & x > 1 \end{Bmatrix}$ is everywhere differentiable, the value of $a + b$ is
(A) 3 (B) 5 (C) -2 (D) 8

18. The value of x for which $A = \begin{bmatrix} 1+x & 7 \\ 3-x & 8 \end{bmatrix}$ is singular is
(A) $\frac{13}{15}$ (B) $\frac{-13}{15}$ (C) $\frac{15}{13}$ (D) none of these

For questions 19 and 20, two statements are given – one labeled Assertion (A) and the other labeled Reason (R), select the correct answer to these questions from the codes (A), (B), (C) and (D) as given below:

(A) Both A and R are true and R is the correct explanation of the assertion
(B) Both A and R are true and R is not the correct explanation of the assertion
(C) A is true, but R is false
(D) A is false, but R is true

19. **Assertion (A) :** Consider the function $f(x) = |x| + |x - 1|, x\epsilon R$, then f is not differentiable at $x = 0$ and $x = 1$.

Reason (R) : Suppose f be defined and continuous on (a, b) and $c\ \epsilon(a, b)$, then f is not differentiable at $x = c$ if $\lim_{h \to 0^-} \frac{f(c+h)-f(c)}{h} \ne \lim_{h \to 0^+} \frac{f(c+h)-f(c)}{h}$

20. **Assertion (A) :** The principal value of $tan^{-1}(-\sqrt{3})\ is\ \frac{-\pi}{3}$.

Reason (R) : We know that for any $x\epsilon R$, $tan^{-1}x$ represents an angle in $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ whose tangent is x.

Contd…3

Section – B

21. Evaluate $sin^{-1}(sin10) + tan^{-1}(tan(-6))$

22. A balloon which always remain spherical has a variable diameter $\frac{3}{2}(2x + 3)$. Determine the rate of change of volume with respect to x.

23. Differentiate $sin^{-1}\left\{\frac{2x}{1+x^2}\right\}$ with respect to $tan^{-1}x$, $-1 < x < 1$

OR

Differentiate $(cosx)^x$; with respect to x ; $x\epsilon(0, \pi/2)$.

24. If a unit vector $|\vec{a}|$ makes angle $\pi/4$ with $\hat{\imath}$, $\pi/3$ with $\hat{\jmath}$ and an acute $angle\ \theta$ with $\hat{k}$, find θ.

OR

Fina a unit vector perpendicular to the plane ABC where A, B, C are the points $(3, -1, 2)$, $(1, -1, -3)$, $(4, -3, 1)$ respectively.

25. Given $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = 12$, find $|\vec{a} \times \hat{b}|$

Section – C

26. If $y = tan^{-1}x$, find $\frac{d^2y}{dx^2}$ in terms of y alone.

27. A man 2 metres high, walks at a uniform speed of 6 metres per minute away from a lamp post, 5 metres high. Find the rate at which the length of his shadow increases.

28. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 2$ and $|\vec{c}| = 3$. If the projection of $\vec{b}$ along $\vec{a}$ is equal to the projection of $\vec{c}$ along $\vec{a}$ and $\vec{b}, \vec{c}$ are perpendicular to each other, find $|3\hat{a} - 2\hat{b} + 2\vec{c}|$.

OR

Find the length of the perpendicular from the point (1, 2, 3) to the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$

29. Evaluate $\int \frac{dx}{\cos(x-a)\cos(x-b)}$

OR

Evaluate $\int e^{-x} cosx \quad dx$

30. Solve the following LPP graphically:

Minimize $z = 20x + 10y$

Subject to $x + 2y \leq 40$

$3x + y \geq 30$

$4x + 3y \geq 60$

$x \geq 0,\ y \geq 0$

31. Probabilities of solving a specific problem independing by A and B are $\frac{1}{2}$ and $\frac{1}{3}$ respectively. If both try to solve the problem independently, find the probability

(i) problem is solved.

(ii) exactly one of them solves the problem.

OR

An urn contains 4 white and 3 red balls. Find the probability distribution of the number of red balls in a random draw of three balls.

Section – D

32. Using integration, find the area of the region bounded by the following curves after making a rough sketch:

$$y = 1 + |x + 1|\ ,\ x = -3\ ,\ x = 3\ ,\ y = 0\ .$$

33. An amount of ₹ 5,000 is put into three investments at the rate of 6%, 7% and 8% per annum respectively. The total annual income is ₹ 358. If the combined income from the first two investments is ₹ 70 more than the third, find the amount of each investment by matrix method.

Contd…4

34. If $x = sint$ and $y = sinpt$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0$

OR

If $x = a\, sint$ and $y = (cost + \log tan(t/2))$, find $\frac{d^2y}{dx^2}$

35. Find the shortest distance between the lines

$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ ***and*** $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$

OR

Find the image of the point (1, 6, 2) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$. Also, write the equation of the line joining the given point and its image and find the length of segment joining the given point and its image.

Section – E

(Case Study Questions)

36. $P(x) = -5x^2 + 125x + 37500$ is the total profit function of a company, where x is the production of the company.
Based on the above information, solve the following questions:

(i) What will be the production when the profit is maximum?
(ii) What will be the maximum profit?
(iii) Check in which internal the profit is strictly increasing?

OR

When the production in 2 units, what will be the profit of the company?

37. An organization conducted bike race under 2 different categories-boys and girls. In all, there were 250 participants. Among all of them finally three from Category I and two from category 2 were selected for the final race. Ravi forms two sets B and G with these participants for his college project.

Let $B = \{b_1, b_2, b_3\}$, $G = \{g_1, g_2\}$ where B represents the set of boys selected and G the set of girls who were selected for the final race. Ravi decides to explore these sets for various types of relations and functions.

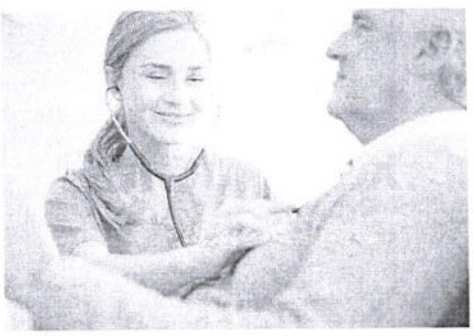
On the basis of the above information, answer the following questions:

(i) Ravi wishes to form all the relatins possible from B to G. How many such relations are possible?
(ii) Write the smallest equivalence relation on G.
(iii) (a) Ravi defines a relation form B to B as $R_1 = \{(b_1, b_2), (b_2, b_1)\}$. Write the minimum ordered pairs to be added in R_1 so that it becomes (A) reflexive but not symmetric, (B) reflesive and symmetric but not transitive.

OR

(iii) (b) If the track of the final race (for the biker b_1) follows the curve $x^2 = 4y$; $(where\ 0 \le x \le 20\sqrt{2}\ and\ 0 \le y \le 200)$, then state whether the track represents a one-one and onto function or not. (Justify):

38. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by cab, metro, bike or by other means of transport are respectively 0.3, 0.2 , 0.1 and 0.4 . The probabilities that he will be late are 0.25 , 0.3, 0.35 and 0.1 if he comes by cab, metro, bike and other means of transport respectively.



On the basis of above information answer the following questions:

(i) When the doctor arrives late, what is the probability that he comes by metro?
(ii) When the doctor arrives late, what is the probability that he comes by other means of transport?

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Class : XII SUBJECT : MATHEMATICS (041) **Max. Marks : 80**

General Instructions:

1. This question paper contains five sections A, B, C, D and E. Each section is compulsory
2. Section – A carries 20 marks weightage, section – B carries 10 marks weightage, section – C carries 18 marks weightage, Section – D carries 20 marks weightage and section – E carries 3 case-based with total weightage of 12 marks.
3. Section – A comprises 20 MCQs of 1 mark each.
4. Section – B comprises 5 VSA type questions of 2 marks each.
5. Section – C comprises 6 SA type of questions of 3 marks each
6. Section – D comprises 4 LA type of questions of 5 marks each.
7. SECTION – E It has 3 case studies. Each case study comprises 3 case-based questions, where 2 VSA type questions are of 1 mark each and 1 SA type question is of 2 marks. Internal choice is provided in 2 marks question in two case-study.
8. Internal choice is provided in 2 questions in section – B, 2 questions in section – C, 2 questions in section – D. You have to attempt only one of the alternatives in all such questions.

SECTION A

1. The domain of $sin^{-1}x + cosx$, is
   (a) $[0,1]$ (b) $[-1,1)$ (c) $[-1,1]$ (d) $[-1, \pi + 1]$
2. For matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} x & 0 \\ 1 & 1 \end{bmatrix}$ and $A = B^2$, then x equals to
   (a) ± 1 (b) -1 (c) 1 (d) 2.
3. If matrix $A = [1 \quad 2 \quad 3]$, AA^T is equal to
   (a) 14 (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix}$ (d) $[14]$
4. If $AB = A$ and $BA = B$ where A and B are square matrices, then
   (a) $B^2 = B$ and $A^2 = A$ (b)) $B^2 \neq B$ and $A^2 = A$ (c)) $B^2 = B$ and $A^2 \neq A$
   (d)) $B^2 \neq B$ and $A^2 \neq A$
5. Let the determinant of a 3×3 matrix A be 6 and B a matrix given by $B = 5A^2$, then $|B| =$
   (a) 750 (b) 180 (c) 270 (d) 4500
6. If $|A| = 2$, where A is 2×2 matrix then $|4A^{-1}| =$
   (a) 4 (b) 2 (c) 8 (d) $\frac{1}{32}$
7. The value of k which makes the function defined by $f(x) = \begin{cases} sin\frac{1}{x}, if\ x \neq 0 \\ k, if\ x = 0 \end{cases}$ continuous at $x = 0$, is
   (a) 8 (b) 1 (c) -1 (d) none of these
8. Side of an equilateral triangle expands at the rate of 2 cm/sec. The rate of increase of its

   area when each side is 10 cm is

   (a) $10\sqrt{2}$ cm^2/sec (b) $10\sqrt{3}$ cm^2/sec (c) 10 cm^2/sec (d) 5 cm^2/sec
9. $\int \frac{1}{x.(x^2+1)} dx$ equals to
   (a) $log|x| - \frac{1}{2}\log(x^2 + 1) + C$ (b) $log|x| + \frac{1}{2}\log(x^2 + 1) + C$ (c) $-log|x| + \frac{1}{2}\log(x^2 + 1) + C$ (d) $\frac{1}{2}log|x| + \log(x^2 + 1) + C$
10. $\int_{-2}^{2} |x \cos \pi x| dx$ is equal to
    (a) $\frac{8}{\pi}$ (b) $\frac{4}{\pi}$ (c)) $\frac{2}{\pi}$ (d) $\frac{1}{\pi}$
11. In The area of the region (in square units) bounded by the curve $x^2 = 4y$, line $x = 2$ and x-axis
    (a) 1 (b) 2/3 (c) 4/3 (d) 8/3
12. The number of arbitrary constants in the particular solution of a differential equation of third order is
    (a) 3 (b) 2 (c) 1 (d) 0
13. The curve for which the slope of the tangent at any point is equal to the ratio of the abscissa to the ordinate of the point is
    (a) an ellipse (b) a parabola (c) a circle (d) a rectangular hyperbola
14. Let $\vec{a}$ and $\vec{b}$ are two vectors inclined at an angle 60°. If $|\vec{a}| = |\vec{b}| = 2$, then the angle between $\vec{a}$ and $\vec{a} + \vec{b}$, is
    (a) 30°. (b) 60°. (c) 45°. (d) none of these .

15. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are any three mutually perpendicular vectors of equal magnitude a, then $|\vec{a}+\vec{b}+\vec{c}|$ is equal to

(a) a (b) $\sqrt{2}\,a$ (c) $\sqrt{3}\,a$ (d) none of these

16. The number of corner points of the feasible region determined by the constraints $x-y \geq 0, 2y \leq x+2, x, y \geq 0$, is

(a) 2 (b) 3 (c) 5 (d) 4

17. The corner points of the feasible region determined by the system of linear constraints are;(0,10),(5,5), (15,15), (0,20). Let $Z = px + qy$, where $p, q > 0$.The condition on p and q so that the maximum of Z occurs at both the points (15,15) and (0,20) is

(a) $p = q$ (b) $p = 2q$ (c) $2p = q$ (d) $3p = q$

18. A random variable X takes the values o,1,2,3 and its mean is 1.3. If $P(X=3) = 2P(X=1)$ and $P(X=2) = 0.3$ then $P(X=0)$ is

(a) 0.1 (b) 0.2 (c) 0.3 (d) 0.4

For questions 19 and 20, two statements are given – one labeled Assertion (A) and the other labeled Reason (R), select the correct answer to these questions from the codes (i), (ii), (iii) and (iv) as given below:

(a) Both A and R are true and R is the correct explanation of the assertion
(b) Both A and R are true and R is not the correct explanation of the assertion
(c) A is true, but R is false
(d) A is false, but R is true

19. Assertion (A) : $f(x) = |x-6| cosx$,is differentiable in $R - \{6\}$
Reason (R) : If a function is continuous at a point c, then it is also differentiable at that point

20. Assertion (A) : The number of reflexive relations on set $A = \{1,2,3,4,5\}$ is 2^{20}

Reason (R) : The number of reflexive relations on a A containing n elements is $2^{n(n-1)}$

SECTION B

21. Evaluate:- $tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + tan^{-1}\left[sin\left(\frac{-\pi}{2}\right)\right]$

22. If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1+logx)^2}$

OR

If $(cosx)^y = (siny)^x$,then prove that $\frac{dy}{dx} = \frac{log\ siny + ytanx}{\log cosx - x\ coty}$.

23. If If$|\vec{a}| = 8, |\vec{b}| = 3, and\ |\vec{a}\times\vec{b}| = 12$ then find $\vec{a}.\vec{b}$.

24. $\vec{a} = 4\hat{\imath} + \hat{2\jmath} - \hat{k}$ and $\vec{b} = 5\hat{\imath} + \hat{2\jmath} - 3\hat{k}$, then find the angle between the vectors $\vec{a}+\vec{b}$ and $\vec{a}-\vec{b}$.

OR

If $|\vec{a}| = 2, |\vec{b}| = 7$ and $\vec{a}\times\vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

25. Prove that function $f(x) = x^3 - 6x^2 + 15x - 18$ is strictly increasing in R

SECTION C

26. Water is leaking from a funnel at the rate of $5cm^3$ /sec. If the radius of the base of the funnel is 10 cm, find the rate at which water level is dropping when it is 5 cm from the top

27. The lengths of three sides of a trapezium other than base are equal to 10 cm each, then find the area of the trapezium when it is maximum.

28. Evaluate:- $\int_0^{\pi/4} log(1+\tan x)\,dx$

29. Find the equation of the line passing through the point $\hat{\imath}+\hat{\jmath}+\hat{k}$ and perpendicular to the lines $\vec{r} = \hat{\imath} + \lambda(2\hat{\imath}+\hat{\jmath}-3\hat{k})$ and $\vec{r} = (2\hat{\imath}+\hat{\jmath}-\hat{k}) + \mu(\hat{\imath}+\hat{\jmath}+\hat{k})$.

OR

Let $\vec{a} = \hat{\imath}-\hat{\jmath},\ \vec{b} = 3\hat{\imath}-\hat{k}$ and $\vec{c} = 7\hat{\imath}-\hat{k}$, find a vector $\vec{d}$ such that $\vec{a}.\vec{d} = 5, \vec{b}.\vec{d} = 7\ and\ \vec{c}.\vec{d} = 1$.

30. Solve linear programming problem graphically, maximize, $Z = 20x + 10y$, subject to constraints $3x + y \leq 24, x + 2y \leq 28,\ x \geq 2, x, y \geq 0$

31. A bag A contains 4 black and 6 red balls and bag B contains 7 black and 3 red balls. A die is thrown. If 1 or 2 appears on it, then bag A is chosen, otherwise bag B. If two balls are drawn at random (without replacement) from the selected bag, Find the probability of one of them being red and another black.

OR

Two numbers are selected at random (without replacement) from the first six positive integers. Let X denotes the larger of the two numbers obtained. Find the probability distribution of the random variable X, and hence find the mean of the distribution.

SECTION D

32. Use the product $\begin{bmatrix}1 & -1 & 0\\ 2 & 3 & 4\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}2 & 2 & -4\\ -4 & 2 & -4\\ 2 & -1 & 5\end{bmatrix}$ to solve the system of equation:- $x - y = 3\ , 2x + 3y + 4z = 17,\ y + 2z = 7$

33. If $x = \tan\left(\frac{1}{a}\log y\right)$, then show that$(1 + x^2)\ \frac{d^2y}{dx^2} + (2x - a)\frac{dy}{dx} = 0$.

OR

If $x = a(cost + t\ sint)$ and $y = a(sint - t\ cost)$,then find $\frac{d^2y}{dx^2}, \frac{d^2y}{dt^2}$ and $\frac{d^2x}{dt^2}$

34. Find the area of the region bounded by the curves $y^2 = x - 2, x = 4\ and\ x = 6$.

OR

Evaluate the area of the region bounded under the curve $\frac{x^2}{4} + \frac{x^2}{9} = 1$and above x-axis.

35. Find the foot of the perpendicular and image of the point P (5,4,2) on the line $\vec{r} = (-\hat{\imath} + 3\hat{\jmath} + \hat{k}) + \mu(2\hat{\imath} + 3\hat{\jmath} - \hat{k})$. Also find the perpendicular distance from the given point to the line.

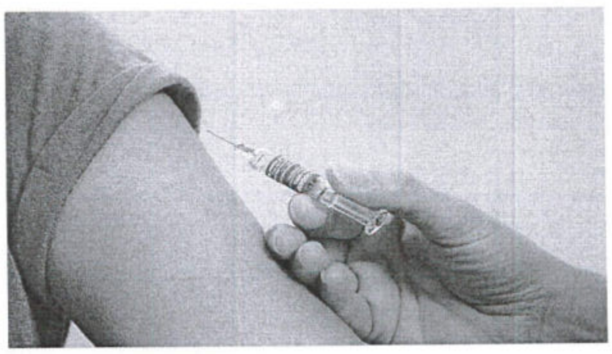
SECTION -E

36. Case Study – 1 CASE STUDY 1: Raji visited the Exhibition along with her family. The Exhibition had a huge swing, which attracted many children. Raji found that the swing traced the path of a Parabola as given by $y = x^2$. Answer the following questions using the above information.
(a) Let $f: R \to R$ be defined by $(x) = x^2$.write the nature of the function
(b) Let $f: N \to N$ be defined by $(x)\ ) = x^2$ is
(c) Let$f : N \to R$ be defined by $(x) = x^2$. Range of the function among the following

37. Case Study – 2 Megha wants to prepare a handmade gift box for her friend's birthday at home. For making lower part of the box, she takes a square piece of cardboard of side 20 cm.
Based on the above, answer the following questions, show steps to support your answer.
(a) If x be the length of each side of the square to be cut off from corners of the square piece of side 20 cm, find the volume of the open box formed (in terms of x) by folding up the cutting corners.
(b) Find the value of x for which $\frac{dV}{dx} = 0$.
(c) Find the maximum volume of the box.
OR,
Find the surface area of the box

38. Case Study-3,Regular blood testing is one of the most effective ways to keep track of your overall physical well-being. It has a variety of uses and is one of the most common types of medical tests done. It helps us to diagnose diseases such as cancer, HIV/AIDS, diabetes, anemia and coronary heart disease.

By total blood count, we can prevent many problems. White blood cells count aware us of any kind of infection and Platelets count aware us of blood clotting issues. Knowing all these can help us to make better decisions for our health. The blood test keeps a record of testosterone and estrogen level in the body



A laboratory blood test is 99% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false-positive result for 0.5% of the healthy person tested (i.e. if a healthy person is tested, then, with probability 0.005, the test will imply he has the disease). If 0.1 per cent of the population actually has the disease,

Based on the above case, answer the following questions.

(a) What is the probability that the person found is corona positive
(b) What is the probability that the person found is corona positive

(c)If in another case, the laboratory blood test is 97% effective in detecting a certain disease when it is in fact, present. However, the test also yields a false-positive result for 0.4% of the healthy person tested. If 0.8 per cent of the population actually has the disease, What is the probability that the person has the disease given that his test result is positive?

-Dewesh Choudhary